APPLICATIONS OF EXPONENTIAL FUNCTIONS

1. $A(30) = 450\left(1 + \dfrac{0.05}{12}\right)^{12\cdot 30} \approx \$2010.48$

2. $5000 = Pe^{0.0225\cdot 10}$ so $P = \frac{5000}{e^{0.0225\cdot 10}} = 5000e^{-0.225} \approx \$3992.58$

3. Skippy invests in an account offers 3.25% interest.

   (a) $2P = P\left(1 + \dfrac{0.0325}{12}\right)^{12t}$. Dividing by P gives $\left(1 + \dfrac{0.0325}{12}\right)^{12t} = 2$.

   Taking natural logs, we get $\ln\left(1 + \dfrac{0.0325}{12}\right)^{12t} = \ln(2)$ or $12t\ln\left(1 + \dfrac{0.0325}{12}\right) = \ln(2)$.

   Hence, $t = \dfrac{\ln(2)}{12\ln\left(1 + \dfrac{0.0325}{12}\right)} \approx 21.36$ years.

   (b) $2P = Pe^{0.0325t}$ so dividing by P gives $e^{0.0325t} = 2$.

   Taking natural logs, we get $\ln\left(e^{0.0325t}\right) = \ln(2)$ or $0.0325t = \ln(2)$.

   Hence, $t = \dfrac{\ln(2)}{0.0325} \approx 21.33$ years. (Not much different than part (a)!)

4. (a) Half life means it takes 25 minutes for half of the substance to decay (so half of it is left!)

   Hence: $\frac{1}{2}A_0 = A_0e^{25k}$. Dividing by A_0 gives $e^{25k} = \frac{1}{2}$.

   Taking natural logs gives $\ln\left(e^{25k}\right) = \ln\left(\frac{1}{2}\right)$ or $25k = \ln\left(\frac{1}{2}\right)$.

   Hence, $k = \dfrac{\ln(1/2)}{25} \approx -0.02772$. Our model is then: $A(t) = A_0e^{-0.02772t}$.

   (b) If 90% decays, only 10% is left. so we solve: $0.1A_0 = A_0e^{-0.02772t}$.

   Dividing by A_0, we get $e^{-0.02772t} = 0.1$.

   Taking natural logs gives $\ln\left(e^{-0.02772t}\right) = \ln(0.1)$ or $-0.02772t = \ln(0.1)$.

   Hence, $t = \dfrac{\ln(0.1)}{-0.02772} \approx 83$ minutes.

5. Here, $T_0 = 180$, $T_a = 72$, so $T(t) = 72 + (180 - 72)\, e^{-kt}$ or $T(t) = 72 + 108e^{-kt}$.

   (a) After 10 minutes, the coffee is 155°F means $T(10) = 155$. Hence: $72 + 108e^{-10k} = 155$.

   Solving this equation for k gives $108e^{-10k} = 83$ or $e^{-10k} = \frac{83}{108}$.

   Taking natural logs gives $\ln\left(e^{-10k}\right) = \ln\left(\frac{83}{108}\right)$ or $-10k = \ln\left(\frac{83}{108}\right)$.

   Hence, $k = -\frac{\ln(83/108)}{10} \approx 0.02633$.

   So we have $T(t) = 72 + 108e^{-0.02633t}$

   (b) To see how long it takes for the coffee to cool to 120°F means solving $T(t) = 72{+}108e^{-0.02633t} = 120$.

   From $72 + 108e^{-0.02633t} = 120$ we get $108e^{-0.02633t} = 48$ or $e^{-0.02633t} = \frac{48}{108} = \frac{4}{9}$.

   Taking the natural log, we get $-0.02633t = \ln\left(\frac{4}{9}\right)$, so $t = -\frac{1}{0.02633}\ln\left(\frac{4}{9}\right) \approx 31$ minutes.

   (c) The horizontal asymptote to the graph of $y = T(t)$ is $y = 72$.

   This means as time goes by, the coffee will cool to room temperature, 72°F

6. (a) $P(0) = \frac{150}{2+73e^{0}} = 2$. This means in 2003, there were just 2 Sasquatch in Roskos Acres.

   (b) Since 2007 is 4 years after 2003, we find $P(4) \approx 12.62$ so between 12 and 13 Sasquatch.

   (c) Solve $P(t) = 50$ means solve $\frac{150}{2+73e^{-0.5t}} = 50$.

   Clearing denominators, we get $150 = 50\left(2 + 73e^{-0.5t}\right)$ or $150 = 100 + 3650e^{-0.5t}$.

   Hence, $3650e^{-0.5t} = 50$ so $e^{-0.5t} = \frac{50}{3650} = \frac{1}{73}$.

   Taking natural logs we get $\ln\left(e^{-0.5t}\right) = \ln\left(\frac{1}{73}\right)$ or $-0.5t = \ln\left(\frac{1}{73}\right)$.

   Hence, $t = -2\ln\left(\frac{1}{73}\right) = 2\ln(73) \approx 8.6$.

   The population will reach 50 Sasquatch sometime during the year 2011.

   (d) The horizontal asymptote of the graph of $y = P(t)$ is $y = 75$.

   This means as time goes by, the Sasquatch Population will approach 75 Sasquatch.